Eur. Phys. J. B **57**, 473–479 (2007)
DOI: 10.1140/epjb/e2007-00181-8

THE EUROPEAN PHYSICAL JOURNAL B

Ising, Schelling and self-organising segregation

D. Stauffer[a] and S. Solomon[b]

Racah Institute of Physics, Hebrew University, 91904 Jerusalem, Israel

Received 2nd May 2007 / Received in final form 5 June 2007
Published online 27 June 2007 –

Abstract. The similarities between phase separation in physics and residential segregation by preference in the Schelling model of 1971 are reviewed. Also, new computer simulations of asymmetric interactions different from the usual Ising model are presented, showing spontaneous magnetisation (=self-organising segregation) and in one case a sharp phase transition.

PACS. 05.50.+q Lattice theory and statistics (Ising, Potts, etc.) – 89.65.-s Social and economic systems – 89.75.-K Complex systems

1 Introduction

More than two millennia ago, the Greek philosopher Empedokles (according to J. Mimkes) observed than humans are like liquids: some mix easily like wine and water, and some do not, like oil and water. Indeed, many binary fluid mixtures have the property that for temperatures T below some critical temperature T_c, they spontaneously separate into one phase rich in one of the two components and another phase rich in the other component. For $T > T_c$, on the other hand, both components mix whatever the mixing ratio of the two components is. Chemicals like isobutyric acid and water, or cyclohexane and aniline, are examples with T_c near room temperature, though they smell badly or are poisonous, respectively. For humans, segregation along racial, ethnic, or religious lines, is well known in many places of the world.

Schelling [1] transformed the Empedokles idea into a quantitative model and studied it. People inhabit a square lattice, where every site has four neighbours to the North, West, South and East. Everyone belongs to one of two groups A and B and prefers to be have neighbours of the same group more than to be surrounded by neighbours of the other group. Thus with some rule depending on the numbers n_A and n_B of neighbours of the two groups, each person moves into a neighbouring empty site. After some time with suitable parameters, large domains were hoped to be formed which are either populated mostly by group A or mostly by group B. A simpler and better version of Jones [2] indeed gave these large domains.

Physicists use the Ising model of 1925 to look at similar effects. Again each site of a large lattice can be A or B or empty; A and B are often called "spin up" and "spin down" in the physics literature referring to quantum-mechanical magnetic moments. The probability to move depends exponentially on the ratio $(n_A - n_B)/T$ calculated from the neighbour states. A B "prefers" to be surrounded by other B, and an A by other A. The lower this temperature or tolerance T is the higher is the probability for A to move to A-rich neighbourhoods, and for B to move to B-rich neighbourhoods. Therefore at low T an initially random distribution of equally many A and B sites will separate into large regions ("domains") rich in A, others rich in B, plus empty regions. In magnetism these domains are called after Weiss since a century and correspond to the ghettos formed in the Schelling model.

This effect can be seen easier without any empty sites. Then either a site A exchanges places with a site B, or a site A is replaced by a site B and vice versa, where in the above probabilities now n_A and n_B are the number of A and B sites in the two involved neighbourhoods. Or, even simpler, a site A changes into a site B or vice versa, involving only one neighbourhood. The latter case can be interpreted as an A person moving into another city, and another person of type B moving into the emptied residence. The physics literature denotes the exchange mechanism as Kawasaki kinetics, the switching mechanism as Glauber (or Metropolis, or Heat Bath) kinetics. Again, at low enough T large A domains are formed, coexisting with large B domains. In the simpler switching algorithm, finally one of these domains wins over the other, and the whole square lattice is occupied by mostly one type, A or B.

The above T can instead of temperature be interpreted socially as tolerance: for high T no such segregation takes place and both groups mix completely whatever the overall composition is. Instead of "tolerance" we may interpret T also as "trouble": external effects, approximated

[a] Visiting from Institute for Theoretical Physics, Cologne University, 50923 Köln, Germany
e-mail: stauffer@thp.uni-koeln.de
[b] e-mail: sorin@cc.huji.ac.il

as random disturbances, may prevent people to live in the preferred residences, due to war, high prices, low incomes, peculiarities of the location, ... Some of these effects were simulated by Fossett [3]. Without these empty sites, we may also interpret A as one type of liquid and B as the other type, and then have a model for the abovementioned binary liquids which may or may not mix with each other via the Kawasaki exchange of places. Alternatively, we may interpret A as a high-density liquid and B as a low-density vapour and then have a model for liquid-vapour phase transitions: only below some very cold temperature can air be liquefied. The first approximate theory for these liquid-vapour equilibria is the van der Waals equation of 1872.

Thus Schelling could have based his work on a long history of physics research, or a film of computer simulation published in Japan around 1968. But in 1971 Schelling did now yet know this physics history [4] and his model was therefore more complicated than needed and was at that time to our knowledge not yet simulated in the Ising model literature. Schelling did not consider $T > 0$ and at $T = 0$ his model has problems (see below) with creating the predicted segregation. Even today, sociologists [3,5–7] do not cite the physics literature on Ising models (and also ignore [2]. Similarly, physics journals until a few years ago ignored the 1971 Schelling publication [8], though recently physicists extended via Ising simulations the Schelling model to cases with T increasing with time [9] and involving more than two groups of people [10]. However, applications of the Ising model to social questions are quite old [12].

In the following section we point out an artifact in the old Schelling model and a simple remedy for it, coming from the rule how to deal with people surrounded by equal numbers of liked and disliked neighbours. We explain in the next section in greater detail the standard Ising simulation methods using the language of human segregation. Then we present two new models. One takes into account that human interactions, in contrast to particles in physics, can be asymmetric: if a man loves a woman it may happen that she does not love him, while in Newtonian physics actio = –reactio: an apple falls down because Earth attracts the apple and the apple attracts Earth. The other model introduces holes (empty residences) similar to the original Schelling work, with symmetric interactions. Also, we check for sharp transitions and smooth interfaces in a Schelling-type model.

2 Artifact in Schelling model

In Schelling's 1971 model, each site of a square lattice is occupied by a person from group A, or a person from group B, or it is empty. People like to have others of the same group among their eight (nearest and next-nearest) neighbours and require that "no fewer than half of one's neighbors be of the same" group (counting only occupied sites as neighbouring people). Thus, if a person has as many A as B neighbours, then in the Schelling model that person does not yet move to another site. Imagine now the following configuration with 12 people from group B surrounded by A on all sides:

```
A A A A A A A A
A A A A A A A A
A A A B B A A A
A A B B B B A A
A A B B B B A A
A A A B B A A A
A A A A A A A A
A A A A A A A A
```

In this case not a single B has a majority of A neighbours, and all A have a majority of A neighbours. Thus none would ever move, and the above configuration is stable. (Similar artifacts are known from Ising models at zero temperature [11].) One can hardly regard the above configuration as segregation when 8 out of 12 B people have a balanced neighbourhood of four A and four B neighbours each. And this small cluster does not grow into a large B ghetto. Also larger configurations with this property can be invented. In fact, at a vacancy concentration of 30% and starting from a random distribution our simulations gave only small domains, with no major changes after about 10 iterations.

To prevent this artifact one should in the case of equally many A and B neighbours allow with 50 percent probability the person to move to another place; and we will implement such a probabilistic rule later.

3 Ising model

Fossett [3] reviews the explanations of segregation by preference of the individuals or by discrimination from the outside. In Schelling's model [1], preference alone could produce segregation, but in reality also discrimination can play a role. For example, Nazi Germany established Jewish ghettos by force in many conquered cities. A simple Ising model without interactions between people can incorporate discrimination with a field h. We assume that a site which is updated in a computer algorithm is occupied with probability p_A proportional to $\exp(h)$ by a person from group A, and with probability $p_B \propto \exp(-h)$ by a B person. Properly normalized we have

$$p_A = e^h/(e^h + e^{-h}), \quad p_B = e^{-h}/(e^h + e^{-h}) \qquad (1)$$

leading to

$$-M = (e^h - e^{-h})/(e^h + e^{-h}) = \tanh(h) \qquad (2)$$

for the relative difference $M = (N_B - N_A)/N$ of all A and B people in large lattices with N sites. There is no need for any computer simulations in this simple limit without interactions between people. In reality, one may have a discrimination with positive h in one part of the lattice and negative h in the rest of the lattice, leading to segregation by discrimination.

Now we generalize the field to include besides this discrimination h also the interactions of site i with its four nearest neighbours, of which n_A are of type A and $n_b = 4 - n_A$ are of type B:

$$h_i = (n_A - n_B)/T' + h \tag{3}$$

where T' is the tolerance towards neighbours from the other group; now also the probabilities

$$p_A(i) = e^{h_i}/(e^{h_i} + e^{-h_i}),$$
$$p_B(i) = e^{-h_i}/(e^{h_i} + e^{-h_i}) \tag{4}$$

depend on the site i. This defines the standard Ising model on the square lattice; of course many variations have been simulated since around 1960, and theoretical arguments showed $T_c = 2/\ln(1+\sqrt{2}) \simeq 2.2$. Thus for all T' below T_c at $h = 0$ the population separates into large B-rich and A-rich domains with composition $(1 \pm M)/2$, whose size increases towards infinity with time, while for $T' > T_c$ no such "infinitely" large domains are formed. Thus we now define $T = T'/T_c$ such that for $T < 1$ we have segregation and for $T > 1$ we have mixing, at zero field. Schelling starts with random configurations but then uses more deterministic rules, analogous to $T = 0$. However, only for $T < 1$ this spatial separation leads to domains growing to infinity for infinite times on infinite lattices.

For positive h, the equilibrium population always has A as majority and B as minority. If we start with a A majority but make h small but negative, then the system may stay for a long time with an A majority until it suddenly "turns" [3] into a stronger B majority: nucleation in metastable states, like the creation of clouds if the relative humidity exceeds 100 percent (in a pure atmosphere).

(Physicists call the above method the heat bath algorithm; alternatives are the Glauber and the Metropolis algorithms. The choice of algorithms affects how fast the system reaches equilibrium and how one specific configuration looks like, but the average equilibrium properties are not affected. That remains mostly true also if in Kawasaki kinetics these updates of single sites are replaced by exchanging the people on two different sites. In contrast, if the lattice is diluted by adding empty sites as in [1], then the transition T may be different from 1.)

Of course, this Ising model is a gross simplification of reality, but these simplifications emphasise the reasons for spontaneous segregation. As stated on page 210 of Fossett [3]: "*Any* choice to seek greater than proportionate contact with co-ethnics necessarily diminishes the possibility for contact with out-groups and increases spatial separation between groups; the particular motivation behind the choice (i.e., attraction vs. aversion) may be a matter of perspective and in any case is largely beside the point".

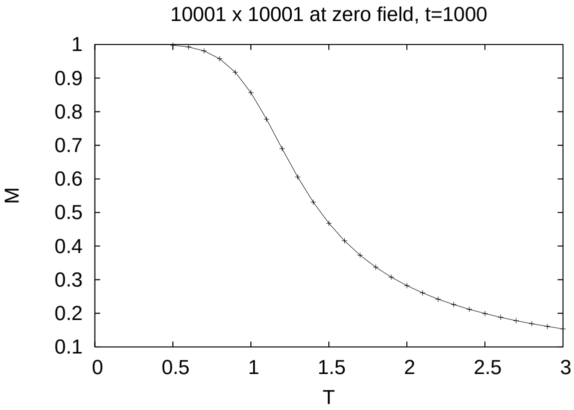

Fig. 1. Composition of the population versus T at $h = 0$, averaged over 1000 sweeps through a lattice of hundred million people.

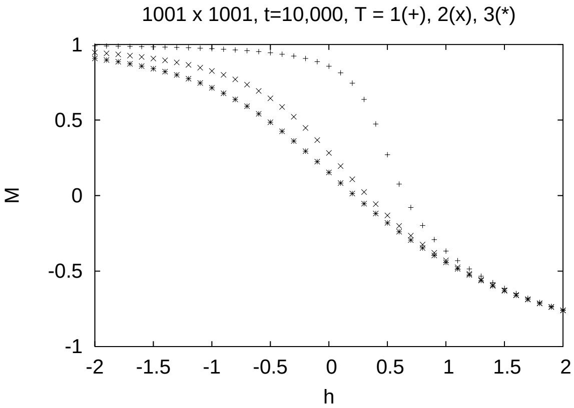

Fig. 2. Composition of the population versus h at fixed $T = 1, 2, 3$, averaged over 10 000 sweeps through a lattice of one million people. This simulation took 4 1/2 h.

4 Modifications

4.1 Asymmetric simulations

In the above model, the rules are completely symmetric with respect to A and B. Fossett [3] reviews the greater willingness of the minority B in American racial relations to mix with the majority A, compared with the willingness of A to accept B neighbours. This we now try to simulate by moving away from physics and by assuming that A is more influenced by B than B is influenced by A. Thus if in the above rule, 3 or 4 of the neighbours are A, then $p_A(i) = p_B(i) = 1/2$. Mathematically, equation (3) is replaced by

$$h_i = \min(0, n_A - n_B)/T + h \tag{5}$$

in our modification. The neutral case of probabilities 1/2 then occurs if A is replaced by B, or B is replaced by A, in a predominantly A neighborhood.

Now the previous sharp transition at $T = 1$, $h = 0$ vanishes: Figure 1 shows smooth curves of M versus T for $h = 0$, and Figure 2 shows smooth curves of M versus h at three fixed T. Maybe this smooth behaviour is judged more realistic by sociology. No segregation into large domains happens, and in contrast to the symmetric Ising model of the preceding section, the results are the same whether we start with everybody A or everybody B.

4.2 Empty spaces

Schelling had to introduce holes (=empty residences) on his lattices since he did now allow a B person to become A or vice versa (via moving to another city) and moved only one person at a time (not letting two people exchange residences). Now we check if holes destroy the sharp transition between self-organised segregation and no such segregation. In physics this is called "dilution", and if the holes are fixed in space one has "quenched" dilution. In this case the fraction of randomly placed holes must stay below 0.407 to give segregation into "infinitely" large domains; for larger hole concentration the lattice separates into fixed finite neighbourhoods of people, separated by holes, such that infinite domains are impossible ("percolation" [13]). For housing in cities, it is more realistic to assume that holes are not fixed: An empty residence is occupied by a new tenant who leaves elsewhere the old residence empty; physicists call this "annealed dilution".

Thus besides A and B sites we have holes (type C) of concentration x, while A and B each have a concentration $(1-x)/2$. People can move into an empty site or exchange residences ("Kawasaki kinetics") with people of the other group, i.e. A exchanges sites with B.

We also replaced the $n_A - n_B$ in equation (3) by the changes in the number of "wrong" neighbours. Thus we calculate the number Δ of A–B neighbour pairs before and after the attempted move, and make this move with a probability proportional to $\exp(-\Delta/T')$; no overall discrimination h was applied. Thus this symmetric model assumes that A does not like to have B neighbours, and B equally does not like A neighbours, while both do not care whether a neighbouring residence is empty or occupied by people of the same group.

Now the total number of A, B and C sites is constant, and a quantity like the above M no longer is useful. Inspection of small lattices shows that again for low T large domains are formed, while for large T they are not formed. To get a more precise border value for T, we let A change into B and B change into A. Then for $T \leq 1.2$ we found that one of the two groups (randomly selected) is completely replaced by the other, while for $T \geq 1.3$ they both coexist.

4.3 Schelling at positive T

Now we simulate a model closer to Schelling's original version, but at $T > 0$, while Schelling dealt with the deterministic motion at $T = 0$. Thus the neighbourhood now includes eight instead of four sites, i.e. besides the four nearest-neighbours we also include the four next-nearest (diagonal) neighbours. Let $n_s(i)$ and $n_d(i)$ be at any moment the numbers of same and different neighbours, respectively, for site i, without counting holes, and let sign be the function sign(k) =1 for $k > 0$, $= 0$ for $k = 0$ and $= -1$ for $k < 0$. A person at site i has an "effort"

$$E_i = \mathrm{sign}[n_d(i) - n_s(i)]. \tag{6}$$

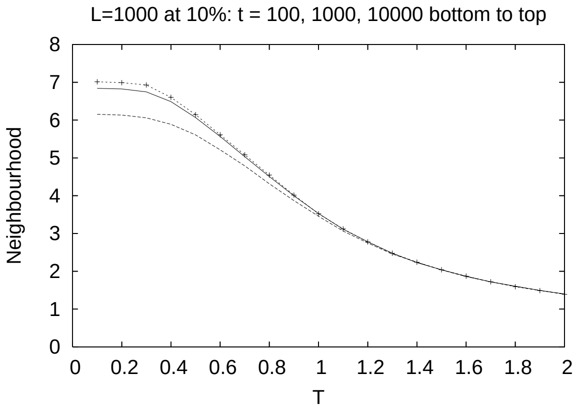

Fig. 3. T dependence of the average number of same minus different neighbours, for three times t showing that about 1000 iterations are enough.

Analogously, E_j is based on the numbers of neighbours of the same and the different type if the person would actually move into residence j. In Schelling's $T = 0$ limit, nobody would move away from i if $E_i < 0$ and nobody would move into an empty site j with $E_j > 0$; instead, people with $E_i > 0$ move into the nearest vacancy j with $E_j \leq 0$.

In reality, one cannot always get what one wants and may have to move into a "bad" neighbourhood. Thus at positive "temperature" T we assume that the move from i to j is made with probability

$$p(i \to j) = e^{-\Delta/T}/(1 + e^{-\Delta/T}) \tag{7b}$$

where

$$\Delta = E_j - E_i \tag{7b}$$

is the effort the person at site i needs in order to move to the vacancy at site j. For $\Delta > 0$, higher T correspond to higher probabilities to move against the own wish, while for the Schelling limit $T \to 0$ nobody moves against the own wish. For negative Δ one "gains" effort and is likely to make that move, with a probability the higher the lower T is. For $T = \infty$ or $\Delta = 0$ the probability to move is 1/2. Each person trying to move selects randomly a vacancy from an extended neighbourhood up to a distance 10 in both directions; after ten unsuccessful attempts to find any vacancy the person gives up and stays at the old residence during this iteration. (We no longer distinguish in this subsection between T and T'. Note that E_i is not an energy in the usual physics sense, and thus this model is not of the Ising type.)

Figure 3 shows the average "neighbourhood" $n_s - n_d$, not counting vacancies, for 1000×1000 lattices for $t =$ 100, 1000 and 10 000 iterations (regular sweeps through the lattice) at a vacancy concentration of 10%. Already lattices of size 100×100 agree with Figure 3 apart from minor fluctuations. Figure 4 shows that for low vacancy concentrations one needs longer times: at 1% and $t = 1000$ the results agree with those at 0.1% and $t = 10\,000$. Although for $T \to 0$ our model does not agree exactly

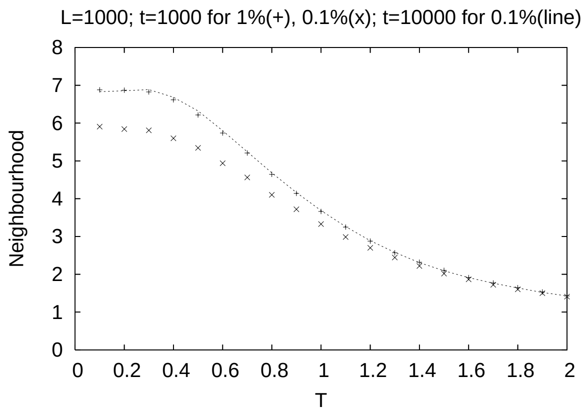

Fig. 4. As Figure 3 but for vacancy concentrations of 0.1 and 1%.

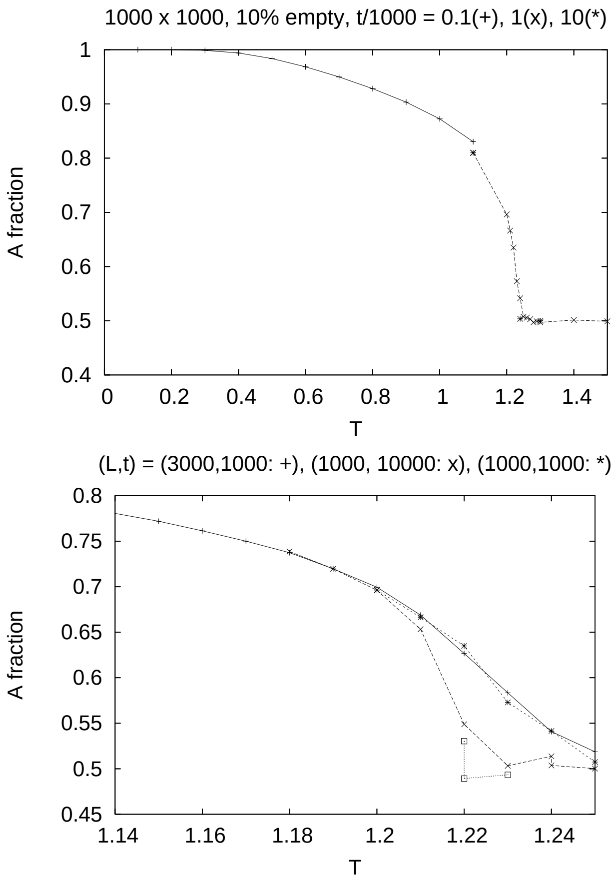

Fig. 5. Spontaneous A-aggregation, i.e. the self-organising degree of segregation $N_A/(N_A + N_B) = (1 - M)/2$ versus T in 1000×1000 lattices after 100 to 10 000 iterations (top). Bottom: additional data up to $t = 10^5$ (squares) close to T_c. 10 percent are vacancies.

with [1] (see Introduction) these figures show clearly the Schelling effect at low T: A becomes surrounded mainly with A neighbours and B with B neighbours, without any outside discrimination. For large T, however, this bias becomes much smaller.

Figure 5 shows the overall fraction of group A (ignoring vacancies) in the interior of large A-rich domains.

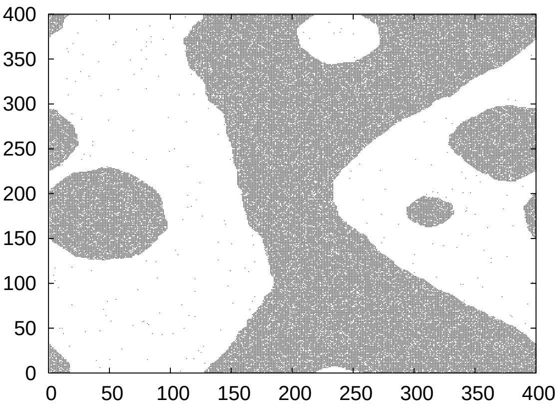

Fig. 6. Distribution of the A population at $T = 0.1$ after 100 000 iterations, showing segregation. 10 percent are vacancies. After 3 million iterations this sample had only one A and one B domain.

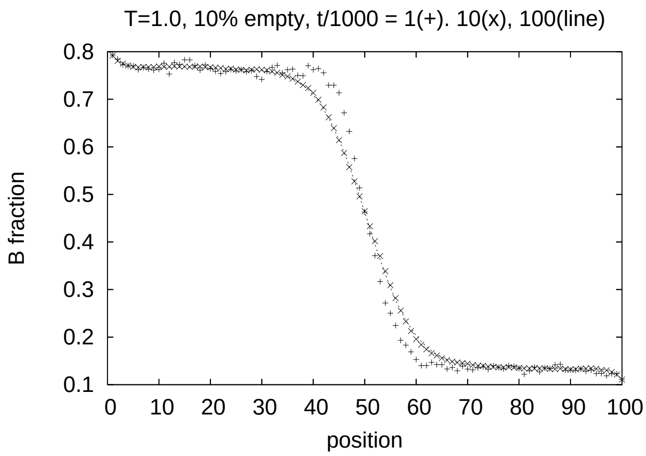

Fig. 7. Profile of the B fraction as a function of position in a 1000×100 lattice, with the interface between A and B domain parallel to the longer side of the rectangle. Averaged over the second half of the simulation.

Figure 6 shows partly the time dependence of segregation, very similar to standard Ising model simulations. For low T we see how very small clusters of A sites increase in size, without yet reaching the size of our 400×400 lattice. (Continuing this simulation to $t = 4$ million gave only one large A domain and one large B domain.) In contrast, for high T these clusters do not grow (not shown). We estimate that near $T = 1.22$ the phase transition occurs between segregating and not segregating conditions, at a vacancy concentration of 10 percent.

Starting in the upper half of the system with one group and in the lower half with the other group, Figure 7 shows for $T < T_c$ how the interface between these to initial domains first widens but then remains limited.

Finally, we also simulate what we believe to be the original Schelling model and we generalise it to positive T. At $T = 0$, each site of a square lattice is occupied by a person from a ("colour") group A, or by one from group B, or is empty. People look at their 8 nearest- and

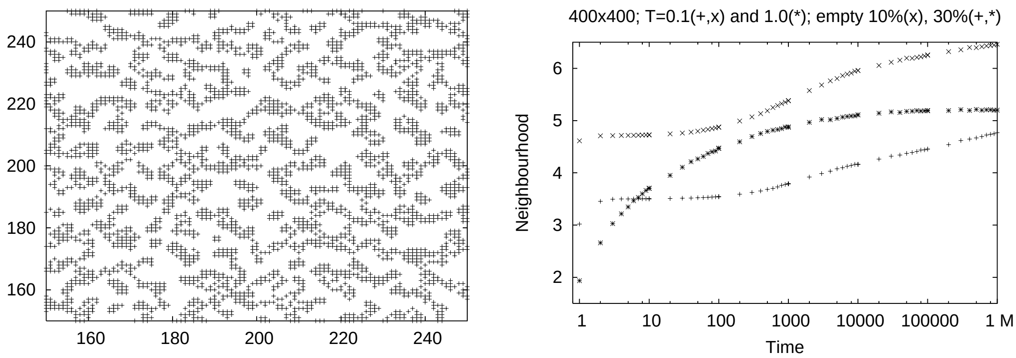

Fig. 8. Left: central part of a 400×400 square lattice with 30% empty sites. The top part represents the configuration obtained after 100 Schelling sweeps through this lattice; important rearrangements happened only during the first 10 sweeps. No large scale segregation occurs. Right: average number of A sites minus number of B sites, in the 8-site neighbourhood surrounding an A site. One observes the slow segregation in the "finite temperature" model for $T = 0.1$ and $T = 1.0$.

next-nearest (diagonal) neighbour sites and if the majority of neighbouring people belongs to the other group, they move into an empty site. The selected empty site is the closest one where the other group does not form the majority of neighbours. Empty sites do not count in determining these majorities. The initial configuration was random.

Vinkovic and Kirman found that only small clusters and no large domains grow [14]. If instead residences are exchanged even if only equally attractive, then they saw large domains. We confirmed both results. Such dependence on whether the attractivity must increase, or merely must not decrease, can be important in simple models [15] but is unrealistic; other aspects of residences which are ignored in a model prevent them to be exactly equally attractive. Our finite temperatures below take care of such minor influences from outside the model. (Ref. [14] erroneously claims that the Ising model changes the colour; see Sect. 3 on Kawasaki kinetics.) Long before, Jones [2] found large domains in a probabilistic modification of Schelling's model, by removing a random fraction of the people and replacing them by people who feel not unhappy on the vacated sites.

Our computer simulations of the above Schelling model, see Figure 8a, show that after <10 iterations, the segregation process stops, even for lattices of $40\,000 \times 40\,000$ sites. (Program from `stauffer@thp.uni-koeln.de`: schelling16.f.) At this stage, the excess of one colour over another in sub-regions of 40×40 of a 400×400 lattice was not distinguishable from the initial random distribution. Thus, the original Schelling model leads to ordering on short length scales but the small clusters of uniform colour do not grow further and there is no significant segregation at lengths above 20. Thus blocking of desegregation cannot only be due to artificial configurations as in Section 2 but occurs automatically in the original Schelling version.

In the "finite temperature" ($T > 0$) generalization of the Schelling model agents move out (with probability $\exp(-1/T)$) even if their present location is acceptable. Moreover, the agents may move (with probability $\exp(-1/T)$) into an undesired location. As seen in Figure 8b, for $T \geq 0.1$, this insures the segregation of the system towards increasingly macroscopic ghettos.

Real ghettos and other forms of racial, ethnic or religious segregation [16] consist of clusters much larger then just a few dozen houses. They involve thousands and perhaps a million people. This large-scale segregation seems not to be modeled by Schelling, and neither he nor recently Fossett [3] seem to have claimed so (Schelling's experiments involved only small numbers of A's and B's).

Thus, the simpler square lattice Ising model leads to more realistic large scale segregation than the more complicated Schelling model. Moreover the Ising model has a simple, analytically tractable extension to "finite temperature" ($T > 0$). The finite temperature amounts to the systematic inclusion of stochastic effects in the deterministic ($T = 0$) model. In particular the "finite temperature" Ising model predicts that the segregation disappears totally and abruptly at a critical value ($T = 2/\ln(1+\sqrt{2})$) in units of the nearest neighbour interaction.

5 Discussion

The similarities between the Schelling and Ising models have been exploited to introduce into the Schelling model the equivalent of the temperature T. This turns out to be a crucial ingredient since it ensures that in the presence of additional random factors the segregation effect can disappear totally in a quite abrupt way. Thus cities or neighbourhoods that are currently strongly polarized may be transformed into an uniformly mixed area by tiny changes in the external conditions: school integration, financial rewards, citizen campaigns, sport centers, common activities, etc. One-dimensional models, like some of Schelling's work, are problematic since at positive T the Ising and many other models do not have a phase transition, while they have one in two and more dimensions.

Besides reviewing the Ising model for non-physicists, we introduced a few modifications to it. Together with those of [9,10] they are only some of the many possible modifications one could simulate. Some confirm the result of Schelling, that even without any outside discrimination, the personal preferences can lead to self-organised segregation into large domains of either mainly A or mainly B people. Other modifications or high T (temperature, tolerance, trouble) prevent this segregation. Thus humans, like milk and honey, are complicated but some of their behaviour can be simulated.

The Schelling model is a nice example how research could have progressed better by more interdisciplinary cooperation between social and natural sciences, and we hope that our paper helps in this direction.

We thank Maxi San Miguel for sending us [4], and A. Kirman, M. Hohnisch, D. Chowdhury and G. Weisbuch for discussion, and K. Müller for telling us about [2,14].

Appendix A: Kinetics

In statistical physics of large systems, including our simulations here, configurations with an energy E are realized with a probability P proportional to $\exp(-E/k_BT)$ where T is the temperature and k_B the Boltzmann constant. With suitable units for T we set $k_B = 1$.For our case here one could call T also the tolerance; it also approximates the many disturbances coming from outside the model. If 1 and 2 denote two possible configurations which can be reached directly from each other, then good algorithms obey the detailed balance principle $R_{12}P_1 = R_{21}P_2$, where R_{12} is the rate to get from state 1 to state 2, and R_{21} is the inverse rate, while P_1 and P_2 are the two equilibrium probabilities. Thus neither Mother Nature nor the algorithms creates a carousel where, for example, the system moves circularly $1 \to 2 \to 3 \to 1 \ldots$ among states of the same energy: "perpetuum mobile". With $\Delta = E_2 - E_1$ we thus need $R_{12}/R21 = \exp(-\Delta/T)$. All algorithms obeying this detailed balance lead sooner or later to the same equilibrium distribution, provided each configuration can in principle be reached from all other configurations, directly or indirectly ("ergodicity").

In our model at finite temperature, the energy for a single person just has two values depending on whether or not the majority of the neighbours belong to the other group, while in the Ising model is has many different values depending on the number of neighbours of each type. We also can move from each configuration in one or several steps to all other configurations, by moving people around on the lattice. Thus the above requirements are fulfilled.

One can distinguish between two main rules determining the kinetics of the process: constant number of people in each group, or fluctuating number of people in each group. The second case can again be separated into Glauber, Metropolis, or Heat Bath variants which is not so important here. Also in the first case, often called the Kawasaki model, various choices obeying detailed balance and ergodicity have been employed for the exchange of a person from group A with one from group B. For example, one may restrict exchanges to nearest neighbours, or let then exchange independent of the geographical distance between them.

In all cases the equilibrium distribution is about the same, while the time needed to reach equilibrium is different. More precisely, for fluctuating populations we may end up with one domain covering the whole lattice, while with constant A and B populations we may have two large domains each covering about a half the lattice. Within each of these two domains, the situation is the same as within the single domain coming from fluctuating populations: each A site has there the same average number of A neighbours etc. Thus Kawasaki kinetics is better to produce nice pictures of moving people while Glauber dynamics etc speeds up the search for phase separation. These kinetic differences are important technically, but are not relevant for the main question whether or not large domains =ghettos are formed. More details can be found in the standard book of Landau and Binder [17], and more simulations in [18].

References

1. T.C. Schelling, J. Math. Sociol. **1**, 143 (1971)
2. F.L. Jones, Aust. NZ. J. Sociol. **21**, 431 (1985)
3. M. Fossett, J. Math. Sociol. **30**, 185 (2006)
4. N.E. Aydinomat, *Schelling*, Economics Bulletin **2**, 1 (2005)
5. W.A.V. Clark, J. Math. Sociol. **30**, 319 (2006)
6. B. Edmonds, D. Hales, J. Math. Sociol. **29**, 209 (2005)
7. J.F. Zhang, J. Math. Sociol. **28**, 147 (2004)
8. M. Levy, H. Levy, S. Solomon, *Microscopic Simulation of Financial Markets* (Academic Press, San Diego, 2000)
9. H. Meyer-Ortmanns, Int. J. Mod. Phys. C **14**, 311 (2003)
10. C. Schulze, Int. J. Mod. Phys. C **16**, 351 (2005)
11. V. Spirin, P.L. Krapivsky, S. Redner, Phys. Rev. E **63**, 036118 (2001)
12. S. Galam, Y. Gefen, Y. Shapir, J. Math. Sociol. **9**, 1 (1982); E. Callen, D. Shapero, Physics Today, July, 23 (1974)
13. D. Stauffer, A. Aharony, *Introduction to Percolation Theory*, 2nd edn. (Taylor and Francis, London, 1992)
14. D. Vinkovic, A. Kirman, Proc. Natl. Acad. Sci. USA **103**, 19261 (2006)
15. D. Stauffer, P.M.C. de Oliveira, Physica A **215**, 407 (1995)
16. J. Friedrichs, Urban Studies **35**, 1745 (1998)
17. D.P. Landau, K. Binder, *A Guide to Monte Carlo Simulations in Statistical Physics* (Cambridge University Press, Cambridge, 2000)
18. K. Müller, C. Schulze, D. Stauffer, e-print `arXiv:0706.2592`